1 Recitation Jan 27,2016 Note

A Few Helping Definitions:
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A) = P(A \cap B) + P(A \cap B^c)$
$Cov(X,Y) = E[(X - E(X))(Y - E(Y))]$
$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$

Law of Iterated Expectations:
$E_X[E[Y|X]] = E[Y]$
Proof:
$E_X[E[Y|X]] =$
$\int [\int yf(y|x)dy]f(x)dx$
$= \int [\int yf(y|x)dy]f(x)dx$
$= \int [\int y\frac{f(y,x)}{f(x)}dy]f(x)dx$
$= \int \int yf(y,x)dydx$
$= \int y \int f(y,x)dxdy$
$= \int yf(y)dy$
$= E[Y]$

Testing Basics:
Step 1: Write Out your null hypothesis. What is it you want to test? Step 2: Under this null, write out a test statistic. Under the null, determine the asymptotic distribution of this test statistic.
Step 3: Plug in values to the test statistic.
Step 4: Using this value, determine whether the test is rejected at a given confidence level and determine the p-value.
P-value interpretation: The probability that you would randomly draw a more extreme value than the observed value from the distribution under the assumption that the null is true.

T-Test Basics:
The basic t-test is $t_\theta = \frac{\hat{\theta}-\theta_0}{SE(\hat{\theta})} \sim N(0,1)$ if n large.
For simple mean: $t_{\bar{X}} = \frac{\bar{X}-\mu_0}{s_X/\sqrt{n}} \sim N(0,1)$ if n large.
For SIMPLE OLS estimator: $t_{\hat{\beta}} = \frac{\hat{\beta}-\beta_0}{SE(\hat{\beta})} \sim N(0,1)$ if n large. $SE(\hat{\beta}) = \sqrt{\frac{var((X_i-\mu_X)u_i)}{n\sigma_X^4}}$
For binomial proportions: $t_{\hat{p}} = \frac{\hat{p}-p_0}{SE(p)} \sim N(0,1)$ if n large. $SE\hat{p} = \sqrt{\frac{p(1-p)}{n}}$
And more... See the "Statistical Hypothesis Testing" wikipedia page – it has a good list. We often do not know the SE of the estimator, so we replace it with an estimate of the SE.