Practice Problems - Weeks 1 through 4

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Question One. Statistical Models

Which of the following describe statistical models?

If it is a statistical model, what are the parameters? What are the parameter spaces?

a) $X_t \sim P$ where $P \in${N(0, 2.3), N(1, 2.3), N(2, 2.3), N(3, 2.3)}

b) $X_t = \beta_0 + \beta_1 X_{t-1} + \epsilon_t$

where $\epsilon_t \sim N(0, \sigma^2)$

c) $E[X_t|Y_t] = 4$

where $Y_t \sim N(2, 4)$

Question Two. Weak Stationarity, Autocovariances.

Which of the following processes are Weakly Stationary? Show a proof.

If they are weakly stationary, derive their autocovariance functions.

a)

$$X_t = .9X_{t-2} + \epsilon_t$$

Where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ iid $\forall t$

b)

$$X_t = .9X_{t-2} + \epsilon_t + .5\epsilon_{t-1}$$

Where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ iid $\forall t$

c)

$$X_t = X_{t-2} + \epsilon_t$$

Where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ iid $\forall t$

d)

$$X_t = t\mu + \epsilon_t + \theta\epsilon_{t-1}$$

Where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ iid $\forall t$

*I invented these questions. All faults are my own. If you have any concerns, queries, or find something that is incorrect, please email me (ndm2125@columbia.edu)

e)

$$X_t = \theta^t exp(\epsilon_t)$$

Where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ iid $\forall t$

Question Two. Stationarity.

a) If a time series process is weakly stationary, is it strongly stationary? Show why using a proof, or why not using a counterexample. [This is a harder question]

b) Take the MA(1) process

$$X_t = \theta_0\epsilon_t + \theta_1\epsilon_{t-1}$$

Where $\epsilon_t \sim N(0, \sigma^2)$

Under what values of $\theta_0, \theta_1, \text{and} \sigma^2$ is the process stationary?

Question Three. Applications of LLN and CLT.

Take the time series process

$$X_t = \phi X_{t-1} + \epsilon_t$$

Where $\epsilon_t \sim N(\mu, \sigma^2)$ iid $\forall t$

a) Suggest a consistent estimator for μ. Which Law of large numbers can you use to prove consistency? Show that its conditions are satisfied.

b) What will be the asymptotic variance of this estimator?

Now take the time series process

$$X_t = \phi X_{t-1} + Y_t$$

Where

$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

And $\epsilon_t \sim N(0, \sigma^2)$ iid $\forall t$

c) Suggest a consistent estimator for μ. Which Law of large numbers can you use to prove consistency? Show that its conditions are satisfied.

d) What will be the asymptotic variance of this estimator?

Question Four. Consistency and Unbiasedness.

Suppose that you have an iid sample $X_1, X_2, ..., X_N$ from a $N(\theta, 1)$ model. Consider the following two estimators. The first one in the sample mean:

$$\hat{\theta}_1 = \frac{1}{N}\sum_{i=1}^{N} X_i$$

The second one is the shrinkage estimator:

$$\hat{\theta}_2 \equiv \frac{1}{2}\hat{\theta}_1$$

For each estimator answer the following questions:

a) Is the estimator consistent? Show this.

b) Is the estimator unbiased? Show this.