Math Camp Recitation Four: Calculus definitions, examples, and intuitions

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1 Announcements

Exam: T/F problems *will* have short proofs/counterexamples.
I will hold a review session at 4PM on Saturday, September 2nd. Location to be determined.
PS 2 to be graded by Thursday.
PS 3 and 4 to be graded by Weekend.

2 Necessary conditions for log linearization method 3

Someone asked what the necessary conditions were for the equation$\hat{}: R \to R$ to apply. I couldn't find any in my searching, but just thinking about it, there are two parts to this question.

First, recall that we defined the log deviation function$\hat{}: \mathbb{R} \to \mathbb{R}$, as:

$$\widehat{f(x)} = log(\frac{f(x)}{f(x_0)})$$

For this to even be calculated, we need the necessary condition that $f(x_0) \neq 0$. Second, we need $log(\frac{f(x)}{f(x_0)})$ to be defined. So, another necessary condition must be that $f : A \to R$ and A is a subset of R (where log is defined on). Finally, log rules are point wise rules. That is, e.g. $log(x/y) = log(x) - log(y) \forall x, y \in R$. Therefore, as long as all functions we are applying are defined on x, the log deviation function rules will apply on x. So, I would write formally, for example:

Let $f : A \to \mathbb{R}$,$g : B \to \mathbb{R}, \alpha \in \mathbb{R}, \beta \in \mathbb{R}$.
If $A \subset \mathbb{R}$,$B \subset \mathbb{R}$, $f(x_0) \neq 0$, and $g(x_0) \neq 0$,
Then, we can define $(f(x)^\alpha \widehat{g(x)^\beta})) : A \cap B \to \mathbb{R}$ as

$$(f(x)^\alpha \widehat{g(x)^\beta})) = \alpha\widehat{f(x)} + \beta\widehat{g(x)}$$

Note: applying the log deviation function is simply algebraic. So, it generally applies when the log rules apply. However, the approximation that $\widehat{f(x)} \approx \frac{f(x)-f(x_0)}{f(x_0)}$ is only good if f(x) is ''close to" $f(x_0)$, as the approximation depends on the Taylor approximation of f(x) around $f(x_0)$. We can guarantee that there exists some neighborhood around x_0 where this is true under an assumption of continuity of f.

3 Normed Vector Space v. Metric Space

What is the difference between a Normed vector space and a metric space?

Hand-written notes - See scanned notes.

4 Applying the Implicit Function Theorem

Hand-written notes - See scanned notes.

5 Leibniz Function usage/examples

The Leibnitz formula states that:
Say $u : A^1_D \to A^1_R, v : A^1_D \to A^1_R$, and $f : A^2 \to \mathbb{R}$ where $A^i \subset \mathbb{R}$. Under a few regularity conditions,[1]

$$\frac{\partial}{\partial x}[\int_{u(x)}^{v(x)} f(x,t)dt] = f(x,v(x))v'(x) - f(x,u(x))u'(x) + \int_{u(x)}^{v(x)} \frac{\partial f(x,t)}{\partial x}dt$$

Why do we care? Often in economics, things are parametrized to account for heterogeneity. For example, selling to many types of markets, time periods, ...

Example: Think of a firm that is selling a good to consumers who are differentiated along some dimension (such as price sensitivity or preference for quality). It cannot differentiate and therefore must choose one price (or some other control) for the good and this price is observed by all. Given a price for the good, each type either buys the good or does not buy the good.

In math: θ- type; p- price

$$\theta \sim U[0,1]$$

$$D(p) = UB(p) - LB(p)$$

[1] A^1_D, A^1_R are closed, $A^1_D \times A^1_R \subset A^2$, u and v are C^1, and f is C^1 in x.

since type θ do not buy if $\theta < LB(p)$ or $\theta > UB(p)$ The firms' profit is then:

$$\Pi(p) = \int_{LB(p)}^{UB(p)} \pi(p,\theta) d\theta$$

Go over graphical intuition here on blackboard.

The first order condition then gives us a formula for the first order condition.

Say $UB(p) = 1 - 1/(p+1)^2$, $LB(p) = 2 * arctan(p)/\pi$, and $\pi(p,\theta) = p(1+\theta)$ Then, equilibria include:

$$\frac{\partial}{\partial p}\Pi(p) = 0$$

What is $\frac{\partial}{\partial p}\Pi(p)$?

$$\frac{\partial}{\partial p}\Pi(p) = \pi(p, UB(p))UB'(p) - \pi(p, LB(p))LB'(p) + \int_{u(p)}^{v(p)} \frac{\partial \pi(p,\theta)}{\partial \theta} d\theta$$

$$\frac{\partial}{\partial p}\Pi(p) = [p(2-1/(p+1)^2)][2/(p+1)^3] - [p(1+2*arctan(p)/\pi)][2/(\pi+\pi p^2)] + \int_{LB(p)}^{UB(p)} p d\theta$$

$$\frac{\partial}{\partial p}\Pi(p) = [p(2-1/(p+1)^2)][2/(p+1)^3] - [p(1+2*arctan(p)/\pi)][2/(\pi+\pi p^2)] + [UB(p) - LB(p)]$$

which is all just a constant with respect to p! This is a much easier problem to solve than before.

6 Next Week

New week I will go over applying and interpreting the Kuhn-Tucker Theorem.