IO Recitation - Oct 10

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1 Differentiated Goods Problems

First, what are differentiated goods? Goods are differentiated if they appeal differently to different types of consumers. There are two types of differentiation:

- Horizontal differentiation
  - Different consumer types prefer different goods if all prices are the same.
  - Examples: *Wall Paint, Political Candidates,Food, Residential Location, Cars,...*
- Vertical differentiation
  - All consumer types prefer the same good if all prices are the same, and agree on the second best good, and the third,....
  - Examples: *Batteries, Rice bags of different quantities,...*

The standard model used to discuss Horizontally Differentiated goods is the Hotelling Line Model.

In the standard Hotelling Line Model:
1) There is a continuum of consumers on a line between 0 and 1.
2) Firms are located on this line in different places. Denote these places L_i.
3) Each of these consumers attain utility v_i from consuming good i. Usually, we assume v_i are all the same, so for the remainder of this class, we will do so and call it v.
4) Each consumer also pays a "travel cost" to consume good i, which varies with how far away they are from firm i. This is usually denoted td where d is the distance. Sometimes we use td^2, but here I will assume linear travel costs.
5) Finally, the consumer pays a price for the good sold by firm i. In short, for a consumer who lives at point x between 0 and 1, their utility from buying a good from firm i is:

$$u_x(i) = v - p_i - t|x - L_i|$$

Consumers buy the good that gives them the highest utility. There are two types of questions I could see Prof. Ho asking regarding differentiated products:

1.1 Fixed Locations, find optimal prices

As in your problem set, she may set the locations of the firm(s), then ask you to find the price that maximizes the firm's profit. The steps to finding this problem are as follows:
STEP ONE: Find the marginal consumers. That is, for each pair of neighboring goods, find the consumer that is indifferent between buying the two. All those to the left will buy the good to the left and all those to the right buy from the firm to the right.
STEP TWO: Construct the demand curve from these marginal consumers. That is, subtract the marginal consumer to the right of each firm to the marginal consumer to the left of each firm. This gives the demand for the good produced by that firm.
STEP THREE: Plug this demand curve into each firm's profit function. Take the first order condition of each firm with respect to their price.
STEP FOUR: Solve the system of equations (system of FOCs) for equilibrium prices.

EXAMPLE:

Say there are 5000 consumers living on Riverside Drive from 100th St to 200th St. It is raining so heavily that everyone must buy an Umbrella. There are 2 umbrella stores on Riverside Dr – Store A at 110th St. and Store B at 150th St. Consumers attain utility $v - p_i - 2d_i$ *from buying the umbrella from store i. Firms have zero marginal cost dollars per umbrella and choose prices simultaneously. At what street will consumers begin to buy from firm B? What will be the equilibrium prices in this market? How many people will buy from A? How many people will buy from B?*

The first thing I would do is to just ignore the extra numbers and solve the model as usual: ie assume they live from 0 to 1, etc. but remember to multiply later!!

STEP ONE: We need to find the marginal consumer between A and B. This consumer is indifferent between buying at each of the stores. To do so, we set the utilities to each other:

$$v - p_A - 2d_A = v - p_B - 2d_B$$

Now, since the marginal consumer will be to the right of A, $d_A = x - L_A = x - .1$, and to the left of B, $d_B = .5 - x$. Our marginal consumer equation becomes:

$$v - p_A - 2(x - .1) = v - p_B - 2(.5 - x)$$

Solving for x:

$$p_B - p_A + .2 + 1 = 4x$$
$$\frac{p_B - p_A}{4} + .3 = x^*$$

STEP TWO: Since all must buy, the demand for A is all those to the left of x^* and the demand for B is all those to the right of x^*.

$$q_A = \frac{p_B - p_A}{4} + .3$$
$$q_A = 1 - \frac{p_B - p_A}{4} + .3 = \frac{p_A - p_B}{4} + .7$$

STEP THREE: Firm A's maximization process is:

$$max_{p_A}(p_A)q_A = (p_A)\frac{p_B - p_A + 1.2}{4}$$

FOC: $\frac{p_B - 2p_A + 1.2}{4} = 0$

$$\frac{p_B + .3}{2} = p_A$$

Firm B's maximization process is:

$$max_{p_B}(p_B)q_B = (p_B)\frac{p_A - p_B + 2.8}{4}$$

FOC: $\frac{p_A - 2p_B + 2.8}{4} = 0$

$$\frac{p_A + .7}{2} = p_B$$

STEP FOUR: Solve the system!

$$\frac{3.5 + .3}{2} = \frac{3}{4}p_A$$
$$p_A = \frac{13}{3}$$
$$p_B = 2.516$$

Finally, transform back its Streets and of consumers.

1.2 Fixed Prices, Choose Location

Example: *Assume that an infinite number of consumers live on a Hotelling line from 0 to 1. The government regulates it so that prices are set equal to 1. Utility of consumers is as usual. Two firms choose their location on the Hotelling line. What will be the Nash Equilibrium locations?*

These questions take a little more thought to answer, but the general strategy is:

1) If they both locate in the center, is there are profitable deviation?

2) If they locate at either end, is there a profitable deviation?
3) If they locate anywhere else, is there a profitable deviation?

As we have seen, the Nash Equilibrium (no profitable deviations) are almost always either option 1 or option 2.

In our example, it is option 1. Why? A firm always has a profitable deviation of moving closer to the other firm, so the only equilibrium is where that deviation is not available: when they are at the same place.

2 Advertising

Two types of Advertising:

- Persuasive advertising: Ads change consumer utility functions. They make consumers enjoy consuming the good more.
- Informative advertising: Ads that inform consumers that a good exists.

How do we model advertising? We assume that both advertising and price effect the amount of goods demanded. That is, we assume that $q_i = f(p, A)$.

From my view, there are two types of questions you could be asked:

2.1 Competitive Advertising with Fixed Prices

In this case, firms will compete against each other by choosing their advertising levels. These question are quite easy: solve them just like the Cournot game, except use advertising as the choice variable instead of quantity.

Example: *Firm A and firm B are competing in a regulated market where prices are set to equal 2 and marginal costs are set to equal 1. Firm A's demand is determined only by its advertising levels:* $q_A = 10 + ln(A_A)$. *Firm B's Demand is determined by the ratio of its advertising and firm A's advertising:* $q_B = 3 + \frac{A_B}{A_A}$. *The firms choose advertising levels simultaneously. Find the equilibrium advertising levels.*

STEP ONE: Plug in demand for good A into good A's profits and take FOC:

$$max_{A_A}(p - c)q_A - A_A = (2 - 1)(10 + ln(A_A)) - A_A$$

And take FOC:

$$\frac{1}{A_A} - 1 = 0 \Rightarrow A_A = 1$$

STEP TWO: Plug in demand for good B into good B's profits and take FOC:

$$max_{A_A}(p-c)q_B - A_B = 3 - \frac{A_A}{A_B} - A_B$$

And take FOC:

$$\frac{A_A}{A_B^2} - 1 = 0 \Rightarrow A_B = \sqrt{A_A}$$

STEP THREE: Solve the system of equations of FOCs. In this example, it is very easy, as A_A does not depend on A_B,but you can see that it could be more complex.

2.2 Monopoly Advertising

This is the example discussed in class. Firms have a demand function that depends on both price and advertising. They simultaneously choose both advertising levels and prices to maximize profits.

Example: *Say a monopolist is in a market where her demand is given by* $q = A^{.8}p^{-1.6}$*, and marginal cost is 1. Find the relationship between equilibrium advertising expenditure and price.*

STEP ONE: plug demand into profit equation:

$$max_{A,p}pq(A,p) - cq(A,p) - A$$

$$max_{A,p}(p-1)A^{.8}p^{-1.6} - A$$

STEP TWO: Take FOCs with respect to p and A:
FOC wrt p:

$$-.6A^{.8}p^{-1.6} + 1.6A^{.8}p^{-2.6} = 0$$

$$(-.6p^{-1.6} + 1.6p^{-2.6})A^{.8} = 0$$

FOC wrt A:

$$.8(p-1)A^{-.2}p^{-1.6} - 1 = 0$$

$$.8(p-1)A^{-.2}p^{-1.6} = 1$$

STEP THREE: Solve the system of equations! Divide FOC1 by FOC2:

$$(-.6 + 1.6p^{-1})A\frac{1}{.8(p-1)} = 0$$

$$\Rightarrow A = \frac{.8(p-1)p}{1.6 - .6p}$$