IO Recitation - Oct 10

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1 Topic One: Solving Cournot Games

General Structure:
Market Demand: P=a-bQ where $Q = q_1 + q_2 + ...$ N firms which may have different marginal costs: $c_1, c_2, c_3...$ Assumptions:
1) Firms Choose their *quantity:* q_i
2) Firms choose *simultaneously*
3) Homogeneous good
4) No dynamics
5) Marginal Costs are public knowledge

Each firm faces the problem:

$$max_{q_i} P(Q)q_i - c_i q_i$$

which can be rewritten as

$$max_{q_i}[a - b(q_1 + q_2 + q_3 + ...)]q_i - c_i q_i$$

What are the strategies for solving these problems?
Strategy One:
IF SYMMETRIC (i.e. $c = c_1 = c_2 = c_3...$)
STEP ONE: Derive first order condition

$$FOC : -bq_i + [a - b(q_1 + q_2 + q_3 + ...)] - c = 0$$

STEP TWO: Assume $q_1 = q_2 = ... = q_i$

$$-bq + [a - bNq] - c = 0$$

STEP THREE: Solve for q:

$$a - b(N+1)q = c$$

$$q = \frac{a-c}{b(N+1)}$$

Strategy Two:
GENERAL CASE: SYMMETRIC OR NON-SYMMETRIC
STEP ONE: Derive first order conditions *for each* firm. For a set of N_1 firms that are the same, just take FOC once.
e.g. for all i,

$$FOC: -bq_i + [a - b(q_1 + q_2 + q_3 + ...)] - c_i = 0$$

STEP TWO: Solve for q_i for all i.

$$q_i = \frac{a-c_i}{2b} - \frac{1}{2}Q_{-i}$$

where $Q_{-i} = \sum_{j \neq i} q_j$ STEP THREE: Add up all FOCs:

$$\sum q_i = \sum \frac{a-c_i}{2b} - \sum \frac{1}{2}Q_{-i}$$

$$Q = \sum \frac{a-c_i}{2b} - \frac{N-1}{2}Q$$

STEP FOUR: Solve for Q:

$$\sum \frac{a-c_i}{2b} = \frac{N+1}{2}Q$$

$$Q = \frac{Na - \sum c_i}{(N+1)b}$$

STEP FIVE: Back out q_i:
From FOC:

$$-bq_i + [a - bQ] - c_i = 0$$

$$q_i = \frac{a-c_i}{b} - Q$$

2 Topic Two: Cournot Example

Lets solve an example using the second method.

Say we have four firms that sell bushels of peaches at a farmers market: two firms are very efficient and have marginal cost of 1; two firms are inefficient and have marginal cost of 4. Demand for apples is given by $P = 50 - 10Q$. The firms compete in Cournot. What is the equilibrium price, market quantity, firm quantities, consumer surplus, and total wellfare?

STEP ONE:
Efficient firms:

$$FOC : -10q_i + [50 - 10(q_1 + q_2 + q_3 + q_4)] - 2 = 0$$

Inefficient firms:

$$FOC : -10q_i + [50 - 10(q_1 + q_2 + q_3 + q_4)] - 10 = 0$$

STEP TWO:
Efficient firms:

$$q_i = \frac{50 - 2}{2(10)} - \frac{1}{2}Q_{-i}$$

$$q_i = 2.4 - \frac{1}{2}Q_{-i}$$

Inefficient firms:

$$q_i = \frac{50 - 10}{2(10)} - \frac{1}{2}Q_{-i}$$

$$q_i = 2 - \frac{1}{2}Q_{-i}$$

STEP THREE: Add up all FOCs:

$$\sum q_i = 2.4 + 2.4 + 2 + 2 - \sum \frac{1}{2}Q_{-i}$$

$$Q = 8.8 - \frac{3}{2}Q$$

STEP FOUR:

$$\frac{5}{2}Q = 8.8$$

$$Q = 3.52$$

Now to find individual quantities:
Efficient firms:

$$FOC : -10q_i + [50 - 10(Q)] - 2 = 0$$

$$-10q_i + 12.8 = 0$$

$$q_i = 1.28$$

Inefficient firms:

$$FOC : -10q_i + [50 - 10(Q)] - 10 = 0$$

$$-10q_i + 4.8 = 0$$

$$q_i = 0.48$$

Now what is price? Plug Q into the demand function:

$$P = 50 - 10Q = 50 - 10(3.52) = 14.8$$

What is consumer surplus? Consumer surplus is area under the demand curve and above the market price.

$$CS = \frac{(50 - 14.8)3.52}{2} = 61.952$$

What is total wellfare? W=CS+PS. Producer surplus is the sum of all profits, so we much first calculate that.

$$\pi_E = (1.28)(14.8 - 2)$$

$$\pi_I = (.48)(14.8 - 10)$$

$$W = CS + 2\pi_E + 2\pi_I$$

3 Topic Three: Problem Set Tips

3.1 Entry with Fixed Costs

In many of the models in this class, a possible question that can be asked on exams or problem sets is ''If there is a fixed cost of F to enter the market, how many firms will enter the market?". To answer this question, use the following methodology:

STEP ONE: Solve the model for individual firm profit after fixed costs that is a function of just N and parameters. That is, $\pi_i(N) = q(N)p(N) - cq(N)$.[1]

STEP TWO: Show that $\pi_i(N)$ is decreasing in N.

[1] For example, in symmetric Cournot with N firms, we found that $q_{ic} = \frac{(a-c)}{(N+1)b}$ and $p_c = a - bNq_{ic}$, and we know that $\pi_i(N) = q_{ic}p_c - cq_{ic}$.

STEP THREE: Solve for the N where $\pi_i(N) = F$. This is the N at which entry is no longer profitable.

STEP FOUR: Round down. This is the equilibrium number of firms in the market.

3.2 Social Planners

In economics, we often talk about social planner problems. In these problems, we choose a parameter to maximize social wellfare under. That is, we cannot change how firms compete once parameters are chosen, but we can choose parameters. This is something that could be a common question on an exam. In general we solve this by:

STEP ONE: Solve for social wellfare as a function of parameters.
STEP TWO: Solve for the parameter values that maximize this social wellfare function. We can do this using first order conditions (if smooth) or with logic (usually if there is a corner solution).

EXAMPLE: Say the social planner can force N inefficient firms to compete in the peach market above, but at a cost of \$10 per firm it forces to do so.

STEP ONE:

$$W(N) = CS(N) + N\pi(N) - 10N$$

$CS(N) + N\pi(N)$ was given in the lecture:

$$CS(N) + N\pi(N) = \frac{(a-c)^2}{2b}\frac{N^2+2N}{N^2+2N+1} = \frac{(50-10)^2}{2(10)}\frac{N^2+2N}{N^2+2N+1} = 80\frac{N^2+2N}{N^2+2N+1}$$

STEP TWO: $max_N W(N)$

$$max_N 80\frac{N^2+2N}{N^2+2N+1} - 10N$$

$$max_N 80(\frac{N^2+2N+1}{N^2+2N+1} - \frac{1}{N^2+2N+1}) - 10N$$

$$max_N 80(1 - \frac{1}{N^2+2N+1}) - 10N$$

$$max_N 80(1 - \frac{1}{(N+1)^2}) - 10N$$

FOC wrt N:

$$80\frac{2}{(N+1)^3} - 10 = 0$$

Solve for N (using Wolfram Alpha):

$$N \approx 1.52$$