IO Recitation - Oct 10

Nathaniel Mark

October 17, 2017

1 Bertrand with Capacity Constraints

Takeaways:
1) Understand why and how to show that Bertrand is not an equilibrium under some capacity constraints.
Say $k_1 = 2, k_2 = 2, MC = 1$ and demand is given by $P = 6 - Q$. Then, setting $p_i = 1$ is not optimal, as $q_i = 2$ given $p_{-i} = 1$ as long as $p_i \leq 2$. Therefore, moving price up is a positive deviation. $\Rightarrow$ Bertrand is not an equilibrium. 2) Know how to set up the problem of setting constraints first: In first stage, each firm chooses k. In second stage, each firm chooses p given k.
3) Know the general solution: The Bertrand equilibrium is mitigated by capacity constraints. That is, prices and profits are increased. 4) Know the specific solution to this model with two firms: *If the cost of capacity is high enough*, then equilibrium of the price competition model with capacity constraints corresponds with the Cournot equilibrium.

Example Question:

2 Dynamic Collusion

Dynamic collusion games ask the question ‘’Is it a Nash Equilibrium for firms to collude?".

Major Assumptions:
1) Firms can coordinate perfectly (as if they were a monopoly)
2) Firms split monopoly profit in collusion (Note: Other assumptions could be made) 3) Firms are infinitely lived
4) Firms can perfectly observe other firm's actions

An equilibrium consists of a set of strategies that offer no positive ‘’One Shot Deviations". What does this mean in practice?

In practice, each firm will have a strategy ‘’Collude if all firms have colluded in the past, but play one-period equilibrium strategy is any firm has cheated anytime in the past." This is called the grim trigger strategy. To test for one shot deviations of this strategy, we must test two things:

1) If other firms are colluding, it is more profitable to collude than to deviate.
2) If other firms are not colluding, it is more profitable to not collude than to collude.

Solving Strategy:
Say you are given problem where two firms compete according to model A in each period and you are asked to solve for the δ such that collusion is sustainable with the usual assumptions. How would you solve this problem?

STEP ONE: Solve the monopoly problem. Attain the split equilibrium profits. Call these $\frac{1}{N}\pi^M$

STEP TWO: Solve the single period model for the static equilibrium. Attain equilibrium profits. Call these π_i^P

STEP THREE: Solve for the maximum profit each firm could attain from deviating in a collusion period. Call this π_i^D.

STEP FOUR: Deviating is not profitable (and therefore collusion is an equilibrium) if

$$\sum_{t=0}^{\infty} \delta^t \frac{1}{N}\pi^M \geq \pi_i^D + \sum_{t=1}^{\infty} \delta^t \pi_i^P$$

Note that since the not-colluding equilibrium is the single period model equilibrium, it is obvious that it is more profitable to not collude than collude then others are not colluding, since not colluding is the *best response*.

2.1 Stackelberg Example

Say that two identical firms with profits $\pi_i = q_i p - 4q_i$ play Stackelberg Cournot. That is, firm 1 chooses quantity, then firm 2 chooses quantity. Total Market Demand is given by $p = 24 - 3(q_{1q2})$. At what δ levels is collusion where monopoly profit is split sustainable under the grim trigger strategy?

STEP ONE: The monopolist faces the following problem:

$$max_Q Q(24 - 3Q) - 4Q$$

FOC:

$$24 - 6Q^* - 4 = 0$$

$$\Rightarrow Q^* = \frac{20}{6}$$

Plugging into profit function:

$$\pi^M = \frac{20}{6}(24 - 3\frac{20}{6}) - 4\frac{20}{6} = \frac{100}{3}$$

STEP TWO: Solving the single-period Stackelberg Game: We solve by *backwards induction*: First, we find what firm 2 will choose given what firm 1 chose already. Firm 2's profit maximization problem is:

$$max_{q_2} q_2(24 - 3(q_1 + q_2)) - 4q_2$$

FOC:

$$24 - 3q_1 - 6q_2^* - 4 = 0$$

$$\Rightarrow q_2^* = \frac{20}{6} - \frac{1}{2}q_1$$

Now, firm 1 knows this is how firm 2 will behave. That is, she knows firm 2's best response function. Therefore, her maximization problem is:

$$max_{q_1} q_1(24 - 3(q_1 + q_2^*(q_1)) - 4q_1$$

We can plug in $q_2^*(q_1)$ that we found above:

$$max_{q_1} q_1(24 - 3(q_1 + \frac{20}{6} - \frac{1}{2}q_1)) - 4q_1$$

Simplifying:

$$max_{q_1} q_1(14 - \frac{3}{2}q_1) - 4q_1$$

FOC:

$$14 - 3q_1^* = 4$$

$$\Rightarrow q_1^* = \frac{10}{3}$$

$$\Rightarrow q_2^* = \frac{20}{6} - \frac{1}{2}\frac{10}{3} = \frac{10}{6}$$

To find profits, we just plug these numbers into profit functions:

$$\pi_2^P = \frac{10}{6}(24 - 3(3\frac{10}{6})) - 4\frac{10}{6} = \frac{25}{3}$$

$$\pi_1^P = \frac{10}{3}(24 - 3(3\frac{10}{6})) - 4\frac{10}{3} = \frac{50}{3}$$

STEP THREE:

Deviating is the same as playing the single period game for firm one, as firm two will observe that they deviated and play their best response accordingly in the same period. So, deviating for firm 1 just means switching to the non-collusion equilibrium. $\Rightarrow \pi_1^D = \frac{50}{3}$

Deviating for the second player: $q_1 = \frac{1}{2}Q^* = \frac{10}{6}$ Therefore, the players maximization process for the deviation period is:

$$\max_{q_2} q_2(24 - 3(\frac{10}{6} + q_2)) - 4q_2$$

FOC solution: (just plugged into BR)

$$\Rightarrow q_2^* = \frac{20}{6} - \frac{1}{2}\frac{10}{6} = \frac{15}{6}$$

Plugging in for profit:

$$\pi_2^D = \frac{15}{6}(24 - 3(\frac{10}{6} + \frac{15}{6})) - 4\frac{15}{6} = 18.75$$

STEP FOUR: Solve for δs such that the no profitable deviation inequality holds.

For firm 1:

$$\sum_{t=0}^{\infty} \delta^t \frac{1}{N}\pi^M \geq \pi_i^D + \sum_{t=1}^{\infty} \delta^t \pi_i^P$$

$$\sum_{t=0}^{\infty} \delta^t \frac{1}{2}\frac{100}{3} \geq \frac{50}{3} + \sum_{t=1}^{\infty} \delta^t \frac{50}{3}$$

$$\frac{1}{1-\delta}\frac{50}{3} \geq (1 + \frac{\delta}{1-\delta})\frac{50}{3}$$

$$\frac{1}{1-\delta} \geq \frac{1}{1-\delta}$$

which always holds.

For firm 2:

$$\sum_{t=0}^{\infty} \delta^t \frac{1}{N}\pi^M \geq \pi_i^D + \sum_{t=1}^{\infty} \delta^t \pi_i^P$$

$$\sum_{t=0}^{\infty} \delta^t \frac{1}{2}\frac{100}{3} \geq 18.75 + \sum_{t=1}^{\infty} \delta^t \frac{25}{3}$$

$$\frac{1}{1-\delta}\frac{50}{3} \geq 18.75 + \frac{\delta}{1-\delta}\frac{25}{3}$$

$$\frac{1}{1-\delta}\frac{50}{3} \geq 18.75 + \frac{\delta}{1-\delta}\frac{25}{3}$$

$$\frac{1-\frac{1}{2}\delta}{1-\delta}\frac{50}{3} \geq 18.75$$

$$\Rightarrow \delta \geq \frac{1}{5}$$

Note: on an exam, writing the second to last step would be fine.

3 Topic Three: Social Planner Problem

In many of the models in this class, a possible question that can be asked on exams or problem sets is ''If there is a fixed cost of F to enter the market, how many firms will enter the market?". To answer this question, use the following methodology:

STEP ONE: Solve the model for individual firm profit after fixed costs that is a function of just N and parameters. That is, $\pi_i(N) = q(N)p(N) - cq(N)$.[1]

STEP TWO: Show that $\pi_i(N)$ is decreasing in N.

STEP THREE: Solve for the N where $\pi_i(N) = F$. This is the N at which entry is no longer profitable.

STEP FOUR: Round down. This is the equilibrium number of firms in the market.

4 Social Planners

In economics, we often talk about social planner problems. In these problems, we choose a parameter to maximize social wellfare under. That is, we cannot change how firms compete once parameters are chosen, but we can choose parameters. This is something that could be a common question on an exam. In general we solve this by:

STEP ONE: Solve for social wellfare as a function of parameters.

STEP TWO: Solve for the parameter values that maximize this social wellfare function. We can do this using first order conditions (if smooth) or with logic (usually if there is a corner solution).

EXAMPLE: Say the social planner can force N firms to compete in the peach market, but at a cost of \$10 per firm it forces to do so. Each firm has marginal cost of 4. Demand for apples is given

[1] For example, in symmetric Cournot with N firms, we found that $q_{ic} = \frac{(a-c)}{(N+1)b}$ and $p_c = a - bNq_{ic}$, and we know that $\pi_i(N) = q_{ic}p_c - cq_{ic}$.

by $P = 50 - 10Q$.

STEP ONE:

$$W(N) = CS(N) + N\pi(N) - 10N$$

$CS(N) + N\pi(N)$ was given in the lecture:

$$CS(N) + N\pi(N) = \frac{(a-c)^2}{2b}\frac{N^2+2N}{N^2+2N+1} = \frac{(50-10)^2}{2(10)}\frac{N^2+2N}{N^2+2N+1} = 80\frac{N^2+2N}{N^2+2N+1}$$

STEP TWO: $max_N W(N)$

$$max_N 80\frac{N^2+2N}{N^2+2N+1} - 10N$$

$$max_N 80(\frac{N^2+2N+1}{N^2+2N+1} - \frac{1}{N^2+2N+1}) - 10N$$

$$max_N 80(1 - \frac{1}{N^2+2N+1}) - 10N$$

$$max_N 80(1 - \frac{1}{(N+1)^2}) - 10N$$

FOC wrt N:

$$80\frac{2}{(N+1)^3} - 10 = 0$$

Solve for N (using Wolfram Alpha):

$$N \approx 1.52$$

Now, since N must be an integer, check whether N=1 or N=2 is better:

$$W(1) = 80(1 - \frac{1}{4}) - 10 = 50$$

$$W(2) = 80(1 - \frac{1}{9}) - 20 = 51.\bar{1}1$$

So, N=2 is optimal.