IO Recitation - Oct 3

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1 Finish Last Week's topic: Equilibrium

1.1 Second degree

- When the monopolist cannot segment the market (cannot observe the type of the consumer)
- Goal: Design a menu to make consumer reveal themselves! This is usually called an adverse selection problem.

Example

The monopolist knows that there is 2 types of consumers in the market: one prefers coffee a lot and one doesn't prefer as much.
For type 1: the utility from drinking q oz of coffee and paying t is $\theta_1 q - t$
For type 2: $\theta_2 q - t$
If $\theta_1 < \theta_2$, which type prefers coffee more ? Assume that the fraction of type 1 is λ, what's the fraction of type 2 ?
Since there are only 2 types, the monopolist would want to design a menu with 2 choices (q_1, t_1) and (q_2, t_2) to maximize his profits.

$$\max_{q_1, t_1, q_2, t_2} \lambda(t_1 - C(q_1)) + (1 - \lambda)(t_2 - C(q_2))$$

There are four constraints:

$$\theta_1 q_1 - t_1 \geq 0$$

This is called individual rationality for low type (IR -L), The left hand side is a low type's consumer's utility from purchasing the the option designed for his type. This condition ensures that the low type consumer would purchase instead of staying out of the market.

$$\theta_2 q_2 - t_2 \geq 0$$

Similarly, individual rationality for high type (IR - H)

$$\theta_1 q_1 - t_1 \geq \theta_1 q_2 - t_2$$

This is incentive compatibility condition for low type (IC-L). The left hand side is the low type's utility from purchasing the option designed for his type (θ_1, q_1), right hand side is the low type's utility from purchasing the option designed for the other type (θ_2, q_2). Remember that the goal is you want to design a menu so that just by looking at which option the consumer picks, you immediately know his type!

$$\theta_2 q_2 - t_2 \geq \theta_2 q_1 - t_1$$

This is incentive compatibility condition for high type (IC-H). Similar intuition as above.

Results when λ is sufficiently big

- IR-L binds: the low type gets zero surplus ("Full Extraction at bottom")
- IC-H binds: the high type is indifferent between buying (θ_1, q_1) and (θ_2, q_2) ("indifference at the top")
- $\theta_2 = C'(q_2)$ (Socially optimal amount sold to high type - no dead-weight loss)
- $C'(q_1) = \frac{1-(1-\lambda)\theta_2}{\lambda} < \theta$ So inefficient amount sold at bottom. That is, a social planner could find a Pareto improving policy by selling one more unit of q_1 to the low types at price θ_1
- the high type retains some surplus (unlike the low type)
- When there are more than two types, the above intuition still holds (Example)
- what happens if λ is small ?

2 Bundling

Main takeaways for bundlings:

- Mixed bundling is always weakly more profitable than pure bundling. (since pure bundling is a subset of mixed bundling)
- Mixed Bundling is always weakly more profitable than no bundling (same reason)
- Pure Bundling is not always more profitable than pure bundling. The details matter: Is WTP high enough compared to marginal cost? Is correlation between WTP_A and WTP_B low enough?

Bundling to Foreclose

We have three firms selling three products X,Y, and Z. People only buy either X and Z together or Y and Z together. How do we show that $(p_X, p_Y, p_Z) = (1, 1, 2)$ is a Nash equilibrium? We must

	Type 1 WTP	Type 2 WTP
XZ	$3 - p_X - p_Z$	$1 - p_X - p_Z$
YZ	$1 - p_Y - p_Z$	$3 - p_Y - p_z$

show that every deviation away from (1,1,2) would yield a lower profit. That is, show that moving p_X to any value below 1 would decrease profit, show that moving p_X to any value above 1 would decrease profit, show that moving p_Y to any value below 1 would decrease profit, show that moving p_Y to any value above 1 would decrease profit, show that moving p_Z to any value below 2 would decrease profit, and show that moving p_Z to any value above 2 would decrease profit. There are many ways to do this.

3 Sub-game Perfect Nash Equilibrium

In an extensive form game, a *subgame* consists of a **single** decision node and **all** its successors in the original game. Each subgame could be played as its own game! Figure one has a circle over every possible subgame in two example games.

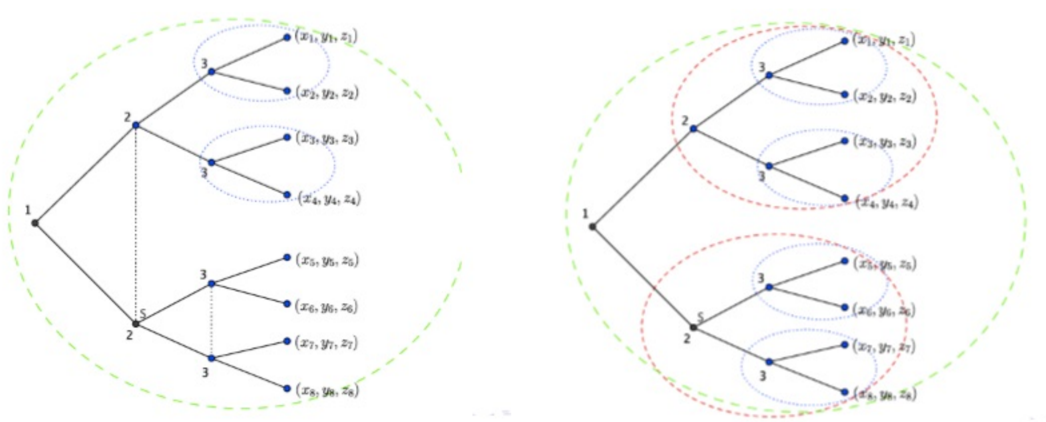

Figure 3.1 – Examples of Subgames

- Now, adding a timing aspect, what would be expect players to rationally do? Perhaps they should assume that all players will be rational (in the Nash sense) in the future. Then, with this assumption, they chose the best response action given those expected future actions. This idea is formalized in the definition of a subgame nash equilibirum.

- A *subgame perfect Nash Equilibrium* is a strategy profile which constitutes a Nash equilibrium in every subgame.
- We solve these problems iteratively:
  Step One: Start with the subgames closest to the terminal nodes. Find the Nash equilibrium of that game.
  Step Two: Act as if the starting node of those solved subgames are now terminal nodes with payoffs corresponding to their Nash Equilibria. Step Three: Repeat steps 1 and 2 until all nodes have an assigned action. This set of actions corresponds with a subgame nash equibirium.
- Remark 1:If any subgame has more than one Nash Equilibrium, you have to do step 2 with each Nash Equilibrium outcome.
- Remark 2: All SPNE are Nash equilibrium. But a Nash equilibrium might not be a SPNE

4 Durable Goods Model

The main durable goods model in our lecture is structured as follows: There are two periods where a monopolist can sell a homogeneous good. Demand is given by $P = \alpha - \beta Q$. One way to think about the model is as follows:

In the second period, the monopolists faces the new demand curve $P = (\alpha - \beta \bar{q}_1) - \beta Q$. Given this demand curve, there is an optimal price/quantity to choose to maximize profit in the second period:

$$p_2 = argmax_p \frac{\alpha - \beta \bar{q}_1 - p}{\beta} p$$

Consumers know this, so for any p_1 chosen, they can do this calculation and therefore have an expectation of p_2.

Therefore, in the first period, the monopolist sets a price p_1. Given that price p_1 and the expectation of the second period price p_2, consumers choose whether to buy in the first period or the second. Those who value the good very highly buy in the first period, those who value it less highly buy it in the second period. The cutoff point on the demand curve where those with low WTP will no longer buy is given by the point at which the "marginal person" is indifferent.

So, our constrained problem in the first period ends up being:

$$max_{p_1,p_2,q_1,q_2} p_1 q_2 + p_2 q_2$$

$$s.t. p_2 = argmax_p \frac{\alpha - \beta q_1 - p}{\beta} p$$

$$q_2 = \frac{\alpha - \beta q_{11} - p_2}{\beta}$$

$$2q_1 - p_1 = q_2 - p_2$$

The first constraint is the subgame nash constraint: The monopolist must choose p_2 to maximize profit in period 2 given the problem they face in period 2. They cant stop themselves from maximizing profit when the time comes.

The second constraint comes by definition of the residual demand curve in the second period.

The third constraint comes from the marginal man condition. It is, in a sense, our first period demand curve. Given a price in period 1, and a price in period 2, the quantity demanded in period 1 is given by this indiference condition.