$$\frac{\left[\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})u_i\right]/\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2}{SE\left[\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})u_i\right]/\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2} \xrightarrow{d} N(0,1)$$

$$\approx \frac{\hat{\beta}_n-\beta_n}{SE(\hat{\beta}_n)} \xrightarrow{d} N(0,1)$$

TOPIC THREE: CONTROL VARIABLES

TRUE MODEL:

$$Y_i=\beta_0+\beta_1 X_{1,i}+\gamma X_{2,i}+u_i$$

We care about β_1, but we do not have data for $X_{2,i}$. This leads to OVB in our estimate $\hat{\beta}_1$

(indeed, recall, $\hat{\beta}_1 \xrightarrow{p} \beta_1 + \underbrace{\text{sign}(\gamma)\,\text{Corr}(X_{1i},X_{2i})\sqrt{\frac{\text{var}(\gamma X_{2i}+u_i)}{\text{var}(X_{1i})}}}_{\text{OVB}}$)

We do have data on W_i, however, and $\text{Corr}(X_{2i}, W_i)=1$. Even though W_i does not directly effect Y_i, running the model

$$Y_i=\beta_0+\beta_1 X_{1i}+\beta_2 W_i+u_i$$

will remove our OVB.