The Math (FOR THE SIMPLE 1 VAR CASE)

UNBIASEDNESS

- START WITH DEFINITION OF β_1

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Before we draw our data, these are all random variables, so note that $\hat{\beta}_1$ is a random variable.

Our next step is to plug in our "true" model: $Y_i = \beta_0 + \beta_1 X_i + u_i$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(\beta_0 + \beta_1 X_i + u_i - (\beta_0 + \beta_1 \bar{X}))}{\sum(x_i - \bar{x})^2}$$

collect terms:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(x_i + \bar{x})\beta_1 + (x_i - \bar{x})u_i}{\sum(x_i - \bar{x})^2}$$

$$= \beta_1 + \frac{\sum_{i=1}^{n}(x_i - \bar{x})u_i}{\sum(x_i - \bar{x})^2}$$

Now, taking expectations to show unbiasedness:

$$\mathbb{E}[\hat{\beta}_1] = \mathbb{E}[\beta_1] + \mathbb{E}\left[\frac{\sum_{i=1}^{n}(x_i - \bar{x})u_i}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right]$$