In short, a variable W_i is a control variable for an omitted variable X_i if:

- $Corr(W_i, X_i)$ is high
- W_i has no direct causal effect on Y_i (the dependent var).

More formally,

under:

1) $\mathbb{E}[u_i | X_i, W_i] = \mathbb{E}[u_i | W_i]$
2) iid data
3) finite 4^{th} moments
4) No perfect multicollinearity,

$\hat{\beta}_1$ is unbiased, $\hat{\beta}_2$ is biased

LSA #1: $\left[\mathbb{E}[u_i | X_{i1}, X_{i2}] = 0\right]$

Looking at this in one more way:

$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + u_i$ is true model.

Since $Corr(X_{i2}, W_i) = 1$, we can write

for some α_0, α_1: $X_{i2} = \alpha_0 + \alpha_1 W_i$

Plugging this in:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 (\alpha_0 + \alpha_1 W_i) + u_i$$

$$= [\beta_0 + \beta_2 \alpha_0] + \beta_1 X_{i1} + \beta_2 \alpha_1 W_i + u_i$$

So, we see this model works, but does not estimate β_0 or β_2, just β_1.