$\hat{\beta}$

$$E[\hat{\beta}_1] = \beta_1 + \mathbb{E}_{\underset{\sim}{x}}\left[\mathbb{E}\left[\frac{\sum_{i=1}^{n}(x_i-\bar{x})u_i}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \,\Big|\, \underset{\sim}{x} \right]\right]$$

by Law of iterated expectations

$$= \beta_1 + \mathbb{E}_{\underset{\sim}{x}}\left[\frac{\sum_{i=1}^{n}(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \underbrace{\mathbb{E}[u_i \mid \underset{\sim}{x}]}_{=0 \text{ by LSA \#1}} \right]$$

$$= \beta_1 \quad \square$$

Note this is where biased estimators come from when this $\neq 0$

Insert (A)

$$\frac{\hat{\beta}_n - \beta_n}{SE(\hat{\beta}_n)} \xrightarrow{d} N(0,1)$$

why? CLT said if V_i is iid & has finite 2^{nd} moments,

$$\frac{\left(\frac{1}{n}\sum_{i=1}^{n} V_i - \mathbb{E}[V_i]\right)}{SE\left(\frac{1}{n}\sum_{i=1}^{n} V_i\right)} \xrightarrow{d} N(0,1)$$

So, to use this, we take

$V_i = (x_i - \bar{x})u_i$ (iid & finite 2^{nd} moment).

$\Rightarrow$ By CLT

$$\frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})u_i - \overbrace{\mathbb{E}[(x_i-\bar{x})u_i]}^{=0}}{SE\left(\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})u_i\right)} \xrightarrow{d} N(0,1)$$